Introduction: Differential Equations are relationship between one or more functions with their derivatives, where the functions generally represents physical quantities and their derivatives represent rates of change. Therefore, differential equations play a wide role in various fields including Engineering, Mechanics, Physics, Biology etc.

Definition of Differential Equation: An equation including dependent variables, independent variables and the derivatives of dependent variables, is called differential equation.

e.g., (i) $\frac{dy}{dx} + y = x$

(ii) $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + y = 5x$

(iii) $6\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} - \frac{dy}{dx} + 3y = x^3$

(iv) $\frac{dy}{dx} + y = \sin y$

(v) $y.\frac{dy}{dx} + \left(\frac{d^2y}{dx^2}\right)^3 = x$

(vi) $\frac{dy}{dx} + y = \sin x$

Note: In above examples, " y " is dependent variable and " x " is independent variable.

Types: Generally, there are two types of differential equations depending on number of independent variables: (1) Ordinary Differential Equations, (2) Partial Differential Equation.

If in a differential equation, number of independent variable is one, then the differential equation is called Ordinary Differential Equation and if number of independent variables are more than one, then the differential equation is called Partial Differential Equation. Examples (i) to (vi) given above are examples of Ordinary Differential Equations. Some examples of Partial Differential Equations are given below:

(i) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$

(ii) $\frac{\partial r}{\partial s} + \frac{\partial^2 r}{\partial t^2} = r$

Note: In this chapter, we will discuss about Ordinary Differential Equations.

Order of Differential Equation: Order of a differential equation is the order of highest derivative exists in differential equation. E.g.,

Differential Equation	Order
(i) $\frac{dy}{dx} + y = x$	1
(ii) $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + y = 5x$	2
(iii) $6\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} - \frac{dy}{dx} + 3y = x^3$	3
(iv) $\frac{dy}{dx} + y = \sin y$	1
(v) $y.\frac{dy}{dx} + \left(\frac{d^2y}{dx^2}\right)^3 = x$	2
(vi) $\frac{dy}{dx} + y = \sin x$	1

Degree of Differential Equation: The degree of differential equation is the power of the highest order derivative in the given differential equation provided that the differential equation satisfies the following conditions:

(i) All the derivatives and dependent variable presented in the differential equation must be free from the fractional powers as well as negative if any.
(ii) There must not be any derivative in denominator of any fraction.
(iii) The highest order derivative must not be argument of any transcendental function, trigonometric function, exponential function, logarithmic function etc.

Note: If one or more of the above conditions are not satisfied by the differential equation, it should be first reduced to the form in which it satisfies all of the above conditions. An equation has no degree or undefined degree if it is not reducible.

E.g.,

Differential Equation	Degree
(i) $\frac{dy}{dx} + y = x$	1
(ii) $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + y = 5x$	1
(iii) $6\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} - \frac{dy}{dx} + 3y = x^3$	1
(iv) $\frac{dy}{dx} + y = \sin y$	1
(v) $y.\frac{dy}{dx} + \left(\frac{d^2y}{dx^2}\right)^3 = x$	3
(vi) $\frac{dy}{dx} + y = \sin x$	1
(vii) $y.\frac{dy}{dx} + \frac{d^2y}{dx^2} = x$	1
(viii) $\sin\left(\frac{dy}{dx}\right) + y = 3$	Undefined

Questions based on Order and Degree of Differential Equations:

Q.1 Find the order and degree of the differential equation

$$\frac{d^2y}{d\,x^2}-\frac{d\,y}{d\,x}+y=x$$

Ans. Order $=2$

Degree $=1$

Note: Observe that the highest order derivative in the differential equation is $\frac{d^2y}{d\,x^2}$, hence the order is 2 and power of $\frac{d^2y}{d\,x^2}$ is 1, so degree is 1.

Q.2 Find the order and degree of the differential equation

$$\left(\frac{d\,y}{d\,x}\right)^5+\left(\frac{d^2y}{d\,x^2}\right)^2+y=\sin x$$

Ans. Order $=2$

Degree $=2$

Note: Observe that the highest order derivative in the differential equation is $\frac{d^2y}{d\,x^2}$, hence the order is 2 and power of $\frac{d^2y}{d\,x^2}$ is 2, so degree is 2.

Q.3 Find the order and degree of the differential equation

$$\left(\frac{d^3y}{d\,x^3}\right)^2+\left(\frac{d^2y}{d\,x^2}\right)^3=e^x$$

Ans. Order $=3$

Degree $=2$

Note: Observe that the highest order derivative in the differential equation is $\frac{d^3y}{d\,x^3}$, hence the order is 3 and power of $\frac{d^3y}{d\,x^3}$ is 2, so degree is 2.

Q.4 Find the order and degree of the differential equation

$$\sin\left(\frac{d^3y}{d\,x^3}\right)-\frac{d^2y}{d\,x^2}+y=x^2$$

Ans. Order $= 3$

Degree is undefined.

Note: Observe that the highest order derivative in the differential equation is $\frac{d^3y}{d\,x^3}$, hence the order is 3 and $\frac{d^3y}{d\,x^3}$ is the argument of sin function, so degree is undefined as the condition (iii) states that the highest order derivative must not be argument of any transcendental function, trigonometric function, exponential function, logarithmic function etc.

Q.5 Find the order and degree of the differential equation

$$\left(\frac{d^2y}{d\,x^2}\right)^{1/2} = \frac{d\,y}{d\,x} + y$$

Ans. Order $= 2$

To find degree, we have to remove the fraction power. So,

$$\left(\frac{d^2y}{d\,x^2}\right)^{1/2} = \frac{d\,y}{d\,x} + y$$

$$\Longrightarrow \qquad \frac{d^2y}{d\,x^2} = \left(\frac{d\,y}{d\,x} + y\right)^2$$

$\Longrightarrow$ Degree $= 1$

Note: Observe that, in the given differential equation is $\frac{d^2y}{d\,x^2}$ has fraction power, hence we first removed the fraction power according to condition (i).

Q.6 Find the order and degree of the differential equation

$$\left(\frac{d^2y}{d\,x^2}\right)^{1/3} = \left(\frac{d^3y}{d\,x^3}\right)^{1/2}$$

Ans. Order $= 3$

To find degree, we have to remove the fraction powers. So,

$$\left(\frac{d^2y}{d\,x^2}\right)^{1/3} = \left(\frac{d^3y}{d\,x^3}\right)^{1/2}$$

$$\Longrightarrow \qquad \left[\left(\frac{d^2y}{d\,x^2}\right)^{1/3}\right]^6 = \left[\left(\frac{d^3y}{d\,x^3}\right)^{1/2}\right]^6$$

$$\Longrightarrow \qquad \left(\frac{d^2y}{d\,x^2}\right)^2 = \left(\frac{d^3y}{d\,x^3}\right)^3$$

$\Longrightarrow$ Degree $= 3$

Note: Observe that, in the given differential equation $\frac{d^2y}{d\,x^2}$ and $\frac{d^3y}{d\,x^3}$ have fraction powers, hence we first removed the fraction powers according to condition (i).

Q.7 Find the order and degree of the differential equation

$$\left(\frac{d^2y}{d\,x^2}\right)^{-2} + y - x = \frac{d^2y}{d\,x^2}$$

Ans. Order $= 2$

To find degree, we have to remove the negative power. So,

$$\left(\frac{d^2y}{d\,x^2}\right)^{-2} + y - x = \frac{d^2y}{d\,x^2}$$

$$\Longrightarrow \quad \frac{1}{\left(\frac{d^2y}{d\,x^2}\right)^2} + y - x = \frac{d^2y}{d\,x^2}$$

$$\Longrightarrow \quad 1 + y\left(\frac{d^2y}{d\,x^2}\right)^2 - x\left(\frac{d^2y}{d\,x^2}\right)^2 = \left(\frac{d^2y}{d\,x^2}\right)^3$$

$\Longrightarrow$ Degree $= 3$

Note: Observe that, in the given differential equation $\frac{d^2y}{d\,x^2}$ has negative power, hence we first removed the negative power according to condition (i).

Q.8 Find the order and degree of the differential equation

$$\frac{1}{\frac{d\,y}{d\,x}} + y = \frac{d\,y}{d\,x}$$

Ans. Order $= 1$

To find degree, we have to remove the derivative from fraction. So,

$$\frac{1}{\frac{d\,y}{d\,x}} + y = \frac{d\,y}{d\,x}$$

$$\Longrightarrow \quad 1 + y\frac{d\,y}{d\,x} = \left(\frac{d\,y}{d\,x}\right)^2$$

$\Longrightarrow$ Degree $= 2$

Note: Observe that, in the given differential equation $\frac{d\,y}{d\,x}$ is in fraction, so we first removed the fraction according to condition (ii).

Exercise:

Q.1 Find the order and degree of the following differential equations:

(i) $\frac{d^2y}{dx^2} - \frac{d^3y}{dx^3} + \frac{dy}{dx} - y = x$

(ii) $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y = \log x$

(iii) $\left(\frac{d^3y}{dx^3}\right)^4 + 5\left(\frac{d^2y}{dx^2}\right)^3 + x^2y = \tan x$

(iv) $e^x\frac{d^4y}{dx^4} - \left(\frac{d^3y}{dx^3}\right)^2 + x\frac{d^2y}{dx^2} = 6x - 2$

(v) $\left(\frac{d^3y}{dx^3}\right)^4 + 8x\left(\frac{d^4y}{dx^4}\right)^3 = x^{12}$

(vi) $6x^3\left(\frac{d^3y}{dx^3}\right)^3 + \left(\frac{d^2y}{dx^2}\right)^2 - x^3y^3 = y^5$

(vii) $\tan\left(\frac{dy}{dx}\right) - 5y = x$

(viii) $\log\left(\frac{d^2y}{dx^2}\right) - \frac{dy}{dx} + y = 3x^2$

(ix) $\left(\frac{dy}{dx}\right)^{1/2} = \frac{d^2y}{dx^2} + y$

(x) $\left(\frac{d^2y}{dx^2}\right)^{1/5} = \left(\frac{d^3y}{dx^3}\right)^{1/4}$

(xi) $\left(\frac{dy}{dx}\right)^{1/2} = x\left(\frac{d^2y}{dx^2}\right)^{1/3}$

(xii) $\left(\frac{dy}{dx}\right)^{-3} + \sin x = \frac{dy}{dx}$

(xiii) $\left(\frac{d^3y}{dx^3}\right)^{-1} + \frac{d^3y}{dx^3} - 6y = \frac{d^2y}{dx^2}$

(xiv) $\frac{1}{\frac{dy}{dx}} + \frac{d^2y}{dx^2} - xy = \frac{dy}{dx}$

(xv) $\left(\frac{d^2y}{dx^2}\right)^4 - \frac{5}{\frac{d^2y}{dx^2}} = \frac{dy}{dx}$

Linear Differential Equation: A differential equation is said to be linear if it satisfies the following conditions:

(i) dependent variable and all its derivatives must occur in first degree,
(ii) dependent variable and all its derivatives should not be multiplied together and
(iii) dependent variable and all its derivatives should not be the argument of another function.

e.g., (1) $\frac{dy}{dx} + y = x$

(2) $6\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} - \frac{dy}{dx} + 3y = x^3$

(3) $\frac{dy}{dx} + y = \sin x$

Note: Here "y" is dependent variable and "x" is independent variable. Observe that all the conditions are applied on dependent variable and on all its derivatives i.e. on "y" and its derivatives. Here "x" is free from all restrictions as independent variable is free from all conditions, i.e., "x" may have any power, it may be argument of any function, it may be constant etc.

Non-linear Differential Equation: A differential equation is said to be non-linear if it fails to satisfy any of the three conditions given above, i.e., a differential equation is non-linear if it doesn't satisfy the conditions of linearity.

e.g., (1) $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + y = 5x$

(2) $y.\frac{dy}{dx} + \frac{d^2y}{dx^2} = x$

(3) $\frac{dy}{dx} + y = \sin y$

Note: Reasons of non-linearity:

In example (1) $\frac{dy}{dx}$ occurs in second power, which is contradiction to condition (i).

In example (2) y is multiplied with $\frac{dy}{dx}$, which is contradiction to condition (ii).

In example (3) y is the argument of sin function, which is contradiction to condition (iii).

Questions based on Linear and Non-linear Ordinary Differential Equations:

Q.1 Check whether the following differential equations are linear or non-linear:

(i) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = x$

(ii) $\left(\frac{dy}{dx}\right)^5 + \left(\frac{d^2y}{dx^2}\right)^2 + y = \sin x$

(iii) $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} = e^y$

(iv) $x^3\frac{d^3y}{dx^3} - x^2\frac{d^2y}{dx^2} + y = 3$

(v) $\frac{dy}{dx} + y = \sin x$

(vi) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4y = \cos y$

(vii) $\frac{d^2y}{dx^2} - \log\left(\frac{dy}{dx}\right) + 4y = x$

(viii) $6 + y.\frac{dy}{dx} = x$

(ix) $y^{\frac{1}{3}} + \frac{dy}{dx} = 0$

(x) $\frac{d^2y}{dx^2} + 6x\left(\frac{dy}{dx}\right)^{-2} + y = x$

Ans.

(i) Linear

(ii) Non-linear

(iii) Non-linear

(iv) Linear

(v) Linear

(vi) Non-linear

(vii) Non-linear

(viii) Non-linear

(ix) Non-linear

(x) Non-linear

Note:

1(ii), 1(ix) and 1(x) are non-linear due to powers of dependent variable and its derivatives.

1(iii), 1(vi) and 1(vii) are non-linear due to functions applied on dependent variable and its derivatives.

1(viii) is non-linear due to multiplication of dependent variable with its derivative.

1(i), 1(iv) and 1(v) are linear because dependent variable and its derivatives satisfy all the conditions of linearity.

Exercise:

Q.1 Check whether the following differential equations are linear or non-linear:

(i) $\frac{d^2y}{d\,x^2}.\frac{d\,y}{d\,x}+y=x$

(ii) $x^2\frac{d^2y}{d\,x^2}-x\frac{d\,y}{d\,x}+y=2$

(iii) $\frac{d^3y}{d\,x^3}+x\frac{d^2y}{d\,x^2}=\sqrt{x}$

(iv) $\frac{d^2y}{d\,x^2}+7x\frac{d\,y}{d\,x}=\sqrt{y}$

(v) $x^3\frac{d^3y}{d\,x^3}+\frac{d^2y}{d\,x^2}+6y=e^{2x}$

(vi) $\frac{d^2y}{d\,x^2}-\frac{d\,y}{d\,x}+y=e^{2y}$

(vii) $\frac{d\,y}{d\,x}-x^4y=\tan x$

(viii) $\frac{d\,y}{d\,x}-\frac{d^2y}{d\,x^2}=\cot y$

(ix) $\frac{d^2y}{d\,x^2}-3\frac{d\,y}{d\,x}+4y=\operatorname{cosec} y$

(x) $\frac{d^2y}{d\,x^2}+\log x\frac{d\,y}{d\,x}+4y=x$

(xi) $\frac{d^2y}{d\,x^2}+\frac{d\,y}{d\,x}+\log y=0$

(xii) $y.\frac{d^2y}{d\,x^2}-\frac{d^2y}{d\,x^2}+5y=x+3$

(xiii) $y+\sqrt{\frac{d\,y}{d\,x}}=1$

(xiv) $y+\left(\frac{d\,y}{d\,x}\right)^{3/2}=1$

(xv) $\left(\frac{d^2y}{d\,x^2}\right)^{-1}+\frac{d\,y}{d\,x}=1-x$

Formation of Ordinary Differential Equations:

Consider the equation

$$f\ (x, y, k) = 0 \qquad (1)$$

where k is the arbitrary constant.

To form the differential equation from this equation, differentiate the equation (1) with respect to the independent variable occur in the equation.

Eliminate the arbitrary constant k from (1) and its derivative. Then we get the required differential equation.

Note: The order of the differential equation to be formed is equal to the number of arbitrary constants present in the equation of the family of curves.

Questions based on Formation of Ordinary Differential Equations:

Q.1 Find the differential equations of the family of curves $y = mx + c$ when

(i) m is the arbitrary constant.

(ii) c is the arbitrary constant.

Ans.

(i) Given family of curves is

$$y = mx + c \qquad (1)$$

where m is the arbitrary constant.

Differentiating equation (1) with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(mx + c)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}(mx + c)$$

$$\Rightarrow \quad \frac{dy}{dx} = m \qquad (2)$$

To find the differential equation, we will have to eliminate arbitrary constant m from equation (1) and equation (2).

Therefore using equation (2) in equation (1), we have

$$y = x\frac{dy}{dx} + c$$

which is required differential equation.

(ii) Given family of curves is

$$y = mx + c \qquad (1)$$

where c is the arbitrary constant.

Differentiating equation (1) with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(mx + c)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}(mx + c)$$

$$\Rightarrow \quad \frac{dy}{dx} = m \qquad (2)$$

In equation (2), we can see that the arbitrary constant c is eliminated.

Hence equation (2) is the required differential equation.

Q.2 Find the differential equations of the family of curves $y = \frac{p}{x} + q$ where p is the arbitrary constant.

Ans. Given family of curves is

$$y = \frac{p}{x} + q \qquad (1)$$

where p is the arbitrary constant.

Differentiating equation (1) with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{p}{x} + q\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}(p\,x^{-1}) + \frac{d}{dx}(q)$$

$$\Rightarrow \quad \frac{dy}{dx} = -p\,x^{-2} \qquad (2)$$

To find the differential equation, we will have to eliminate arbitrary constant p from equation (1) and equation (2).

From equation (1), we have

$$p = x(y - q)$$

Using this value of p in equation (2), we have

$$\frac{dy}{dx} = -x(y-q)\,x^{-2}$$

$$\Rightarrow \qquad \frac{dy}{dx} = -(y-q)\,x^{-1}$$

which is required differential equation.

Q.3 Find the differential equations of the family of curves $y = a\sin x$ where a is the arbitrary constant.

Ans. Given family of curves is

$$y = a\sin x \qquad (1)$$

where p is the arbitrary constant.

Differentiating equation (1) with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(a\sin x)$$

$$\Rightarrow \qquad \frac{dy}{dx} = a\cos x \qquad (2)$$

To find the differential equation, we will have to eliminate arbitrary constant a from equation (1) and equation (2).

From equation (1), we have

$$a = \frac{y}{\sin x}$$

Using this value of a in equation (2), we have

$$\frac{dy}{dx} = \frac{y}{\sin x}\,.\cos x$$

$$\Rightarrow \qquad \frac{dy}{dx} = y.\cot x$$

which is required differential equation.

Q.4 Find the differential equations of the family of curves $y = ae^{4x} + b$ where a is the arbitrary constant.

Ans. Given family of curves is

$$y = ae^{4x} + b \qquad (1)$$

where a is the arbitrary constant.

Differentiating equation (1) with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(ae^{4x} + b)$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{d}{dx}(ae^{4x}) + \frac{d}{dx}(b)$$

$\Rightarrow \qquad \frac{dy}{dx} = 4ae^{4x}$ (2)

To find the differential equation, we will have to eliminate arbitrary constant a from equation (1) and equation (2).

From equation (1), we have

$$ae^{4x} = y - b$$

Using this value in equation (2), we have

$$\frac{dy}{dx} = 4(y - b)$$

which is required differential equation.

Variable Separable Method: A differential equation of the from

$$\frac{dy}{dx} = f(x, y) \qquad (1)$$

is said to separable equation if $f(x, y)$ can be written in the form

$$f(x, y) = p(x).q(y)$$

and then we can separate the variables to solve the given equation as

$$\frac{dy}{dx} = f(x, y)$$

$$\Rightarrow \qquad \frac{dy}{dx} = p(x).q(y)$$

$$\Rightarrow \qquad \frac{dy}{q(y)} = p(x)dx$$

Now integrating both sides, we get

$$\int \frac{dy}{q(y)} = \int p(x)dx$$

$$\Rightarrow \qquad Q(y) = P(x) + c \qquad (2)$$

where $Q(y) = \int \frac{dy}{q(y)}$, $P(x) = \int p(x)dx$ and c is constant of integration.

Equation (2) is the required solution of equation (1).

Questions based on Variable Separable Method:

Q.1 Solve the following differential equations:

(i) $\frac{d\,y}{d\,x} = x^2$

(ii) $\frac{d\,y}{d\,x} = 2x + \sin x$

(iii) $x^{-6}\frac{d\,y}{d\,x} = 5$

(iv) $\frac{d\,y}{d\,x} = x.\log x$

(v) $\frac{d\,y}{d\,x} = e^{x-y}$

(vi) $\frac{d\,y}{d\,x} = xy + y + x + 1$

(vii) $\frac{d\,y}{d\,x} = \frac{\sin x}{\cos}$

Ans.

(i) Given that

$$\frac{d\,y}{d\,x} = x^2$$

Separating the variables, we get

$$d\,y = x^2 dx$$

Now, integrating both sides, we get

$$\int d\,y = \int x^2 dx$$

$$\Rightarrow \quad y = \frac{x^3}{3} + c$$

which is the required solution of given differential equation.

(ii) Given that

$$\frac{d\,y}{d\,x} = 2x + \sin x$$

Separating the variables, we get

$$d\,y = (2x + \sin x)dx$$

Now, integrating both sides, we get

$$\int d\,y = \int (2x + \sin x)dx$$

$\Rightarrow \qquad y = 2\frac{x^2}{2} - \cos x + c$

$\Rightarrow \qquad y = x^2 - \cos x + c$

which is the required solution of given differential equation.

(iii) Given that

$$x^{-6}\frac{d\,y}{d\,x} = 5$$

Separating the variables, we get

$$d\,y = 5\,x^6\,dx$$

Now, integrating both sides, we get

$$\int d\,y = \int 5\,x^6\,dx$$

$\Rightarrow \qquad y = \frac{5\,x^{6+1}}{6+1} + c$

$\Rightarrow \qquad y = \frac{5\,x^7}{7} + c$

which is the required solution of given differential equation.

(iv) Given that

$$\frac{d\,y}{d\,x} = x.\log x$$

Separating the variables, we get

$$d\,y = x.\log x\,dx$$

Now, integrating both sides, we get

$$\int d\,y = \int x.\log x\,dx$$

$\Rightarrow \qquad y = \log x \int x\,dx - \int \left[\frac{d}{dx}(\log x).\int x\,dx\right] dx + c$

(used product rule of integration)

$\Rightarrow \qquad y = \log x.\frac{x^2}{2} - \int \left[\frac{1}{x}.\ \frac{x^2}{2}\right] dx + c$

$\Rightarrow \qquad y = \frac{x^2\ \log x}{2} - \int \frac{x}{2}\,dx + c$

$\Rightarrow \qquad y = \frac{x^2\ \log x}{2} - \frac{1}{2}.\ \frac{x^2}{2} + c$

$\Rightarrow \qquad y = \frac{x^2\ \log x}{2} - \ \frac{x^2}{4} + c$

which is the required solution of given differential equation.

(v) Given that

$$\frac{dy}{dx} = e^{x-y}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{x}.e^{-y}$$

Separating the variables, we get

$$\frac{dy}{e^{-y}} = e^{x}dx$$

$$\Rightarrow \quad e^{y}dy = e^{x}dx$$

Now, integrating both sides, we get

$$\int e^{y}d\,y = \int e^{x}dx$$

$$\Rightarrow \quad e^{y} = e^{x} + c$$

which is the required solution of given differential equation.

(vi) Given that

$$\frac{dy}{dx} = xy + y + x + 1$$

$$\Rightarrow \quad \frac{dy}{dx} = y(x+1) + 1(x+1)$$

$$\Rightarrow \quad \frac{dy}{dx} = (x+1)(y+1)$$

Separating the variables, we get

$$\frac{dy}{(y+1)} = (x+1)dx$$

Now, integrating both sides, we get

$$\int \frac{dy}{(y+1)} = \int (x+1)dx$$

$$\Rightarrow \quad \log|y+1| = \frac{x^2}{2} + x + c$$

which is the required solution of given differential equation.

(vii) Given that

$$\frac{dy}{dx} = \frac{\sin x}{\cos y}$$

Separating the variables, we get

$$\cos y\, dy = \sin x\, dx$$

Now, integrating both sides, we get

$$\int \cos y \; dy = \int \sin x \, dx$$

$\Rightarrow$ $$\sin y = -\cos x + c$$

which is the required solution of given differential equation.

Exercise:

Q.1 Solve the following differential equations:

(i) $x\frac{d\,y}{d\,x} = 4$

(ii) $\frac{d\,y}{d\,x} = 5x^4 + \tan x$

(iii) $\frac{d\,y}{d\,x} = \frac{y}{x}$

(iv) $\frac{d\,y}{d\,x} = x.\cos x$

(v) $\frac{d\,y}{d\,x} = e^{3y+x}$

(vi) $\frac{d\,y}{d\,x} = xy + 2y + 2x + 4$

(vii) $\frac{d\,y}{d\,x} = \frac{\cos x}{\sec y \; tan\; y}$

(viii) $\frac{d\,x}{d\,y} = \frac{5\; cosec^2 y - 1}{3^x}$

(ix) $\frac{d\,y}{d\,x} = e^{x-y} + x^2 e^{-y}$

(x) $\frac{d\,x}{d\,y} = \frac{2}{x^2\; e^{-5y}}$